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On the application of successive plane strains to grid-generated turbulence

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A grid-generated turbulence is subjected to a pure plane strain and the principal axes of the Reynolds stress tensor become those of the strain. This 'oriented' homogeneous turbulence is then submitted to a new strain the principal axes of which have a different orientation. We show that in such a situation it is possible to observe a transfer of energy from the fluctuating motion to the mean one. Such transfer is necessarily associated with a forced decay of the anisotropy of the motion. A detailed analysis of the reorientation of the principal axes of the Reynolds stress tensor in the frame of those of the second strain gives an explanation of the evolution of the principal axes of the Reynolds stress tensor in a shear flow.

1. Introduction

If we consider that the description of a turbulent flow can be made by splitting all instantaneous quantities into an averaged and a fluctuating part, it is clear that understanding of the physical process begins in particular with a detailed analysis of the action of the fluctuating motion on itself and the action of the mean motion on the turbulent field. Since all these different interactions occur simultaneously in a real flow, it is logical to try to separate them and, for this purpose, to build a hierarchy of physical models in which the turbulent motion is simpler than in real turbulent flows. The hierarchy begins with isotropic turbulence in which only the fundamental nonlinear mechanism of transfer between wave vectors is retained. This is one of the aspects of the action of the fluctuating motion on itself. Another feature of this nonlinear self interaction can be found in the second step in the hierarchy which is the model of homogeneous but non-isotropic turbulence with no mean velocity gradient (Batchelor 1960). In that case there appears a strong tendency to a return to isotropy which has been recently analysed in detail by Lumley & Newman (1977). Since no mean velocity gradient exists in the two previous models, the kinetic energy of the fluctuating motion is always decaying.

The next step in the hierarchy is to consider the action of the mean motion on the fluctuating one. Most patterns of flow exhibit such a situation; one of the simplest is the model of homogeneous turbulence associated with a constant mean velocity gradient which has been extensively developed by Craya (1958).

The simplest experiment concerning this model is the action of a constant pure plane strain on an initially isotropic turbulence (Townsend 1954; Tucker & Reynolds 1968; Maréchal 1970). In that case the principal axes of the Reynolds stress tensor are those of the mean rate of strain and the turbulent motion appears as 'oriented' in

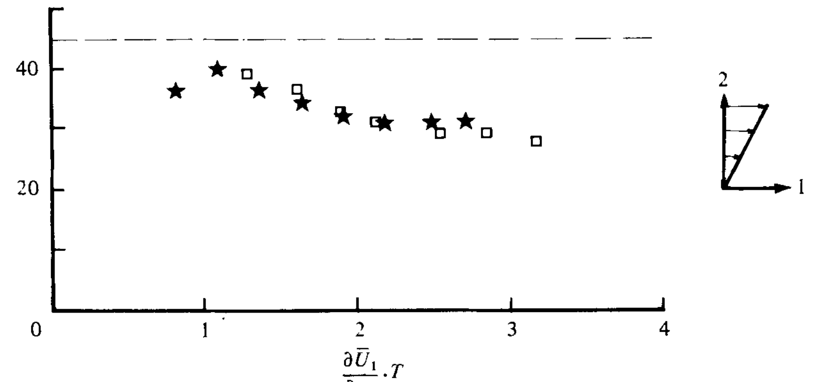

FIGURE 1. Evolution (in degrees) of the angle of the principal axes of the Reynolds stress tensor in a shear flow as a function of the non-dimensional product $(\partial \bar{U}_1/\partial X_2)\, t$. ——, Angle of the principal axes of the rate of strain tensor. ★, from Rose (1966); □, from Champagne, Harris & Corrsin (1970).

these axes. It must be noticed that of course, in such a physical model, the action of the mean velocity gradient occurs simultaneously with the previous nonlinear mechanisms, but it is always possible to neglect the latter during a finite time interval if the mean gradient is large enough, as in the 'sudden distortion' of Batchelor & Proudman (1954). A second fundamental flow of this kind, which has been experimentally studied, is the action of a constant shear on an initially isotropic turbulence (Rose 1966; Champagne, Harris & Corrsin 1970; Harris, Graham & Corrsin 1977). In that case, the mean gradient is compounded of a pure plane strain associated with a mean rotation. Its action on the fluctuating motion is equivalent to one of a pure plane strain the principal axes of which are instantaneously turning around an axis perpendicular to the plane of the strain. The experiment shows (figure 1) that in that case the principal axes of the Reynolds stress tensor are not aligned with those of the strain, which is undoubtedly a consequence of the mean rotation. In particular, we can conclude that, in a shear flow, the associated plane strain is always acting on a fluctuating motion in which the principal axes of the Reynolds stress tensor are never aligned with those of the strain.

Disregarding the problem of the mean rotation, we report in this paper on an experimental study of a pure plane strain acting on a turbulence in which the principal axes of the Reynolds stress tensor are not initially the same as those of the strain. This physical situation can be placed between the classical pure plane strain acting on an initially isotropic turbulence and the constant mean shear flow.

2. Experimental procedure

Two problems are to be solved. First we try to obtain a homogeneous turbulent field in which the direction of the principal axes of the Reynolds stress tensor is well determined. In particular we know that, when a pure plane strain is applied to a quasi-isotropic grid turbulence, the principal axes of the Reynolds stress are aligned with those of the strain as shown in figure 2.

Next we try to apply to the quasi-homogeneous turbulence coming from this first strain a second strain whose principal axes have been rotated an angle α in the plane of

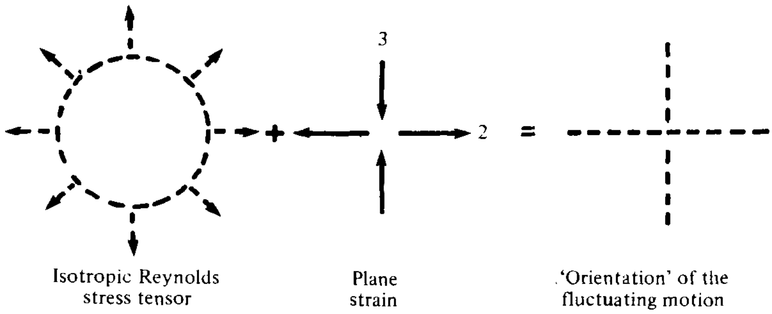

FIGURE 2. For a plane strain applied to a quasi-isotropic grid-turbulence, the principal axes of the Reynolds stress are aligned with those of the strain.

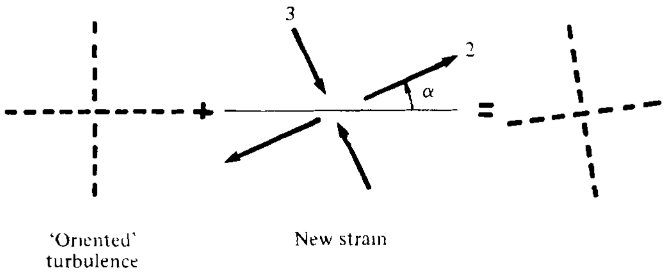

FIGURE 3. The application of a second strain whose principal axes have been rotated at angle α in the plane of the first strain.

the first strain. We represent this phenomenon in figure 3. Of course, the Reynolds stress will be strongly influenced by the new strain and will forget its first orientation with a relaxation time which has to be determined by experiment.

For the design of a distorting duct which realizes the two successive plane strains we will start from the following considerations. We know (Townsend 1954) that a velocity field whose gradient is given in cartesian co-ordinates by the matrix

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & D & 0 \\ 0 & 0 & -D \end{bmatrix}, \tag{1}$$

has streamlines given by

$$x_3 = x_{30} \exp\left(-\frac{D}{U_1} x_1\right), \quad x_2 = x_{20} \exp\left(\frac{D}{U_1} x_1\right). \tag{2}$$

In all previous experiments concerning a pure plane strain the surface of the distorting duct is generated from an initial rectangular section. If, instead, this initial section is elliptic and given by the equation in the plane (x_2, x_3)

$$\frac{x_{20}^2}{a^2} + \frac{x_{30}^2}{b^2} = 1, \tag{3}$$

the surface of the distorting duct which produces the gradient (1), and which is a stream surface, will be given by the equation

$$\frac{x_2^2}{a^2 \exp[(2D/U_1) x_1]} + \frac{x_3^2}{b^2 \exp[-(2D/U_1) x_1]} = 1. \tag{4}$$

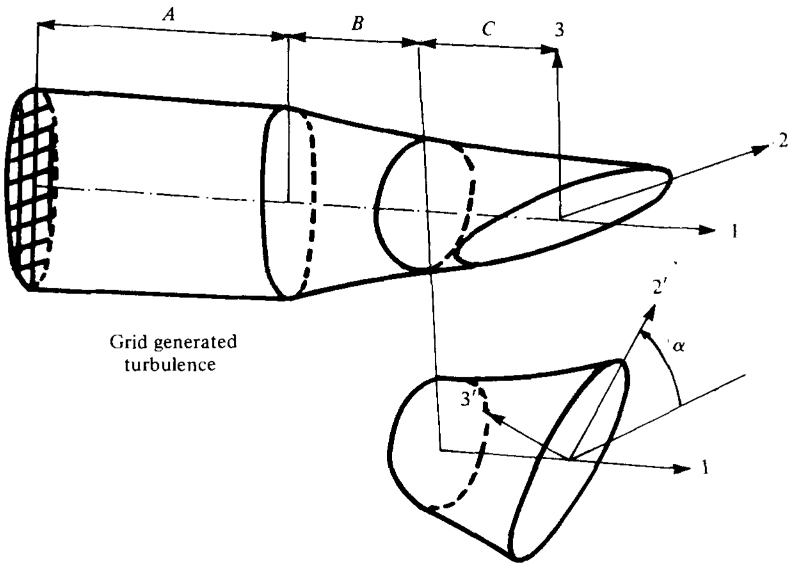

FIGURE 4. The distorting duct and the different frames in which the tensors will be projected.

Then it is easy to see that the cross-section of the duct at x_1 is an ellipse the principal axes of which along x_2 and x_3 are respectively

$$a \exp[(D/U_1)x], \quad b \exp[(D/U_1)x_1].$$

In particular at the value

$$x_1 = \frac{U_1}{2D} \ln \frac{b}{a} = \tfrac{1}{2}L, \tag{5}$$

the cross-section is a circle of radius

$$R = (ab)^{\frac{1}{2}}. \tag{6}$$

Moreover, for a duct of length

$$L = \frac{U_1}{D} \ln \frac{b}{a}, \tag{7}$$

the last section is an ellipse which can be found from the initial one by a rotation of 90° (figure 4). Taking into account the fact that such a duct has a circular cross-section at $x_1 = \frac{1}{2}L$, we can cut the duct into two parts B and C as shown in figure 4 and turn the part C an angle α around the x_1 axis. In part A a quasi isotropic turbulence is generated from a grid possessing a square 3·5 cm mesh, which is located at 40 mesh lengths behind the entrance of the distorting duct. In part B, the fluctuating motion is 'oriented' by the first strain as indicated in figure 2 and in part C the second strain acts on this initially 'oriented' turbulent motion as is shown in figure 3.

The total length L of the distorting duct is 0·8 m and the principal axes of the elliptic initial section are respectively 0·3 m and 0·075 m. All measurements are made with standard DISA hot-wire probes and with constant temperature DISA anemometers 55 M01 associated with DISA units 35 D25. The measurements of the correlation $\overline{u_2 u_3}$ when it exists in the plane of the strain were obtained by use of a method derived from

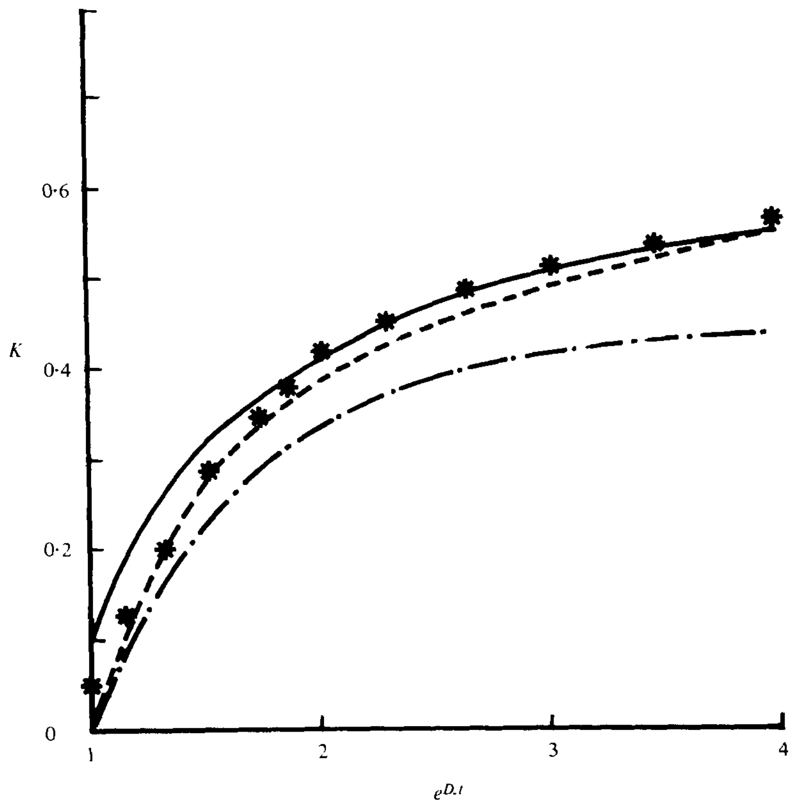

FIGURE 5. Evolution in the case $\alpha = 0$ of the ratio $k = (\overline{u_3^2}-\overline{u_2^2})/\overline{u_3^2}+\overline{u_2^2})$ as a function of the strain ratio e^{Dt}. Comparison of results: —·—, Townsend (1954); ---, Tucker & Reynolds (1968); ——, Maréchal (1970); ✱, present results.

the one proposed by Fugita & Kovasznay (1968). The axial velocity is equal to 18·6 m/s, so that the intensity of the strain is 32·23 s^{-1}. The trace of the Reynolds stress tensor at the entrance of the duct is $\overline{q_0^2} = 0{\cdot}43\ m^2\ s^{-2}$.

3. Experimental results and comments

We give in this part the evolution of the components of the Reynolds stress tensor for different values of α: 0, $\frac{1}{8}\pi$, $\frac{1}{4}\pi$, $\frac{3}{8}\pi$, $\frac{1}{2}\pi$. All the results are plotted *versus* x_1/L and Dt, or e^{Dt}, which is the strain ratio. We know that in the case of a sudden distortion (Batchelor & Proudman 1954; see also Courseau 1974) the components of the Reynolds stress appear as functions of this exponential quantity.

Of course, for $\alpha = 0$, that is to say for the classical case of a pure plane strain, we should recover the results obtained by Townsend (1954), Tucker & Reynolds (1968) and Maréchal (1970). We see in particular in figure 5 that the evolution of the ratio

$$k = (\overline{u_3^2}-\overline{u_2^2})/(\overline{u_3^2}+\overline{u_2^2}) \qquad (8)$$

considered as a function of the strain ratio, agrees well with the results of Tucker & Reynolds (1968) and Maréchal (1970) but a discrepancy with Townsend's results must

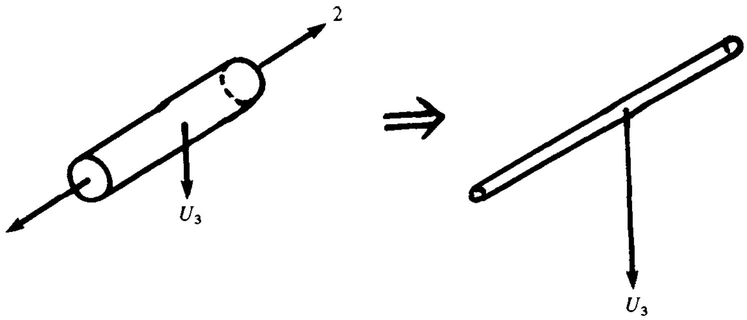

FIGURE 6. Influence of the strain on an eddy aligned with the X_2 axis.

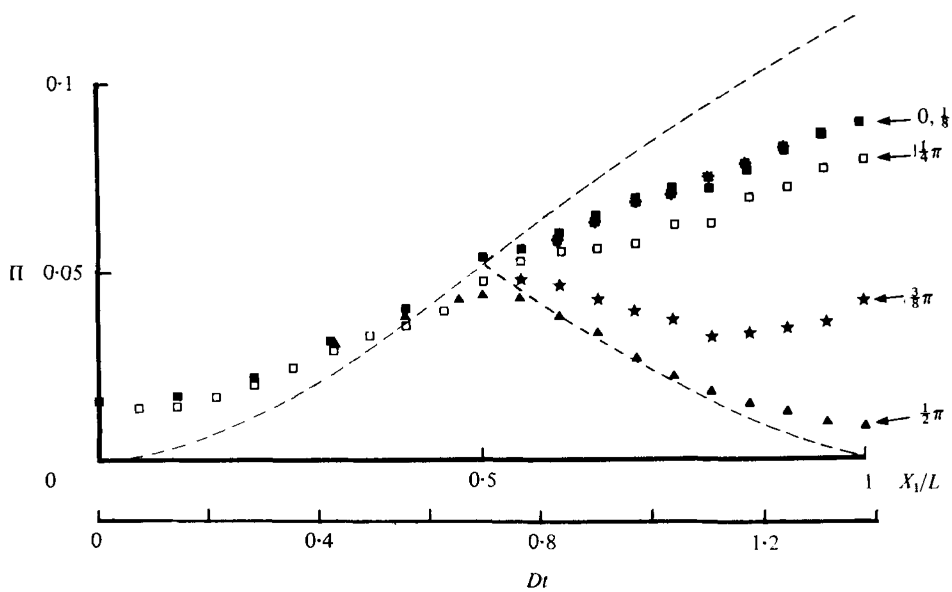

FIGURE 7. Evolution of the invariant Π as a function of X_1/L and Dt for different values of α ($0, \frac{1}{8}\pi, \frac{1}{4}\pi, \frac{3}{8}\pi, \frac{1}{2}\pi$). ---, numerical results obtained from a rapid distortion theory for $\alpha = 0$ and $\frac{1}{2}\pi$ by Boschiero, Gence & Mathieu (1977).

be noticed. As K is not a tensorial quantity we prefer to characterize the anisotropy of the flow by the non-dimensional tensor used by Lumley (1975), which is defined by

$$b_{ij} = \overline{u_i u_j}/\overline{q^2} - \tfrac{1}{3}\delta_{ij}. \tag{9}$$

The invariant

$$\Pi = b_{ik} b_{ki}, \tag{10}$$

which appears as a good scalar measure of the state of anisotropy of the turbulence, plays a similar role to K. In the case $\alpha = 0$, a very simple explanation of the growth of K can be given. Indeed, we can say that all eddies which are aligned with the direction of extension x_2 will turn faster and faster and then induce a component $\overline{u_3^2}$ which will increase (figure 6). Of course with the same reasoning it can be argued that $\overline{u_2^2}$ must decrease. The result is also a growth of the invariant Π as is shown in figure 7. Another consequence is the growth of b_{33} and the decay of b_{22} (figure 8) which has already been

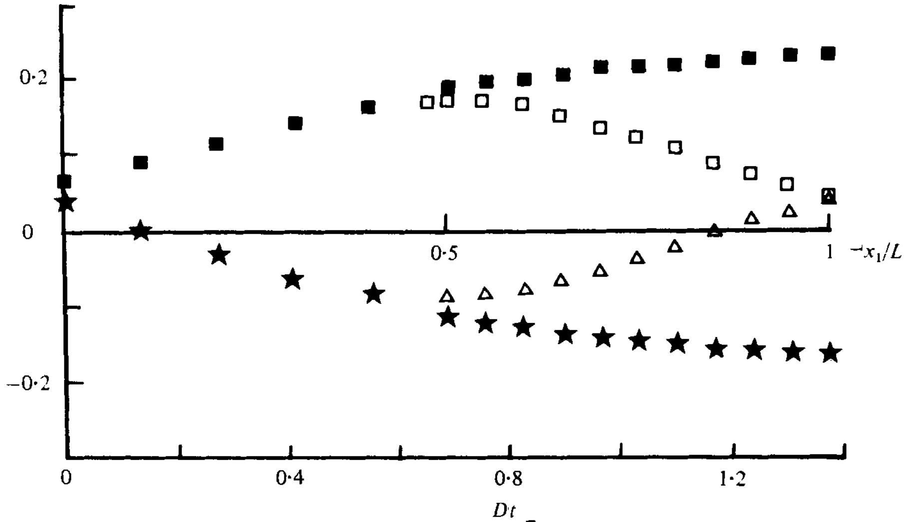

FIGURE 8. Evolution of the components b_{22} and b_{33}. For $\alpha = 0$: ★, b_{22}; ■, b_{33}; for $\alpha = \frac{1}{2}\pi$: △, b_{22}; □, b_{33}.

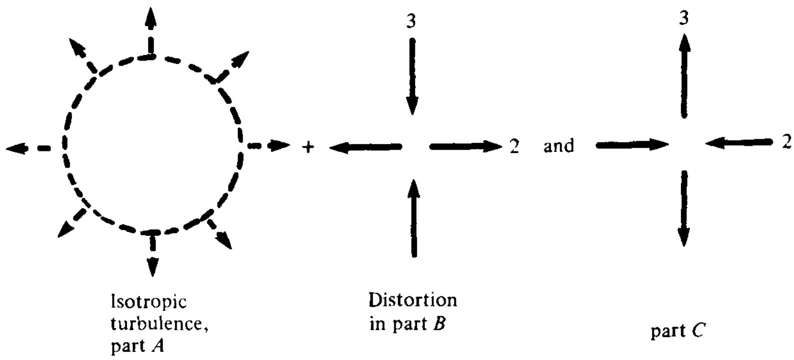

FIGURE 9. The application of the plane strain of opposite signs.

emphasized by Tucker & Reynolds (1968). In particular for the pure plane strain applied to isotropic turbulence, we can write for all values of t

$$b_{33} - b_{22} > 0. \tag{11}$$

Then, in the rate equation for the kinetic energy of the fluctuating motion $\frac{1}{2}\overline{q^2}$

$$\frac{\partial \overline{q^2}}{\partial t} = \underline{-D\overline{q^2}(b_{22} - b_{33})} - 2\bar{\epsilon}, \tag{12}$$

where $\bar{\epsilon}$ is the usual dissipation term. The underlined term, which is the well-known coupling between the mean motion and the fluctuating one, is positive and the mean motion gives energy to the fluctuating one.

This remark may appear as very trivial but will have some importance if we consider the case where $\alpha = \frac{1}{2}\pi$, that is to say, when two plane strains of opposite sign are successively applied as indicated in figure 9. In this case, in part B of the distorting duct, the evolution of $\overline{q^2}$ is given by (12) where the coupling term

$$-D\overline{q^2}(b_{22} - b_{33}) > 0 \tag{13}$$

and in part C, just after the change in sign of the strain,

$$\frac{\partial \overline{q^2}}{\partial t} = +D\overline{q^2}(b_{22}-b_{33})-2\bar{\epsilon}. \tag{14}$$

Hence the coupling term is negative during the whole time when b_{33} is greater than b_{22}. Figure 8 shows that this is the case up to the end of the distorting duct. We can then conclude that the Reynolds stress, which has been subjected to a pure strain of characteristic time $1/D$ during a time T, needs the same time to forget the influence of this strain when it is subjected to a new strain with a same characteristic time but of opposite sign. This phenomenon of reversibility can be understood from Craya's equation (1958) if all the nonlinear effects and the viscosity are neglected. Indeed, under these hypotheses this equation, which gives the evolution of the spatial Fourier transform $\Phi_{ij}(\mathbf{k}, t)$ of the double correlations at two points $\overline{u_i u_j'}$ $(\mathbf{r}, t)$, reduces to

$$\frac{\partial}{\partial t}\Phi_{ij}-\overline{U}_{l,m}\left[k_l\frac{\partial \Phi_{ij}}{\partial k_m}+2\frac{k_i k_l}{k^2}\Phi_{mj}+2\frac{k_j k_l}{k^2}\Phi_{im}-\delta_{il}\Phi_{mj}-\delta_{jl}\Phi_{im}\right]=0. \tag{15}$$

It is then clear that this equation is invariant if the sign of the time t and of the mean velocity gradient are simultaneously changed. This remark gives an explanation of the observed reversibility only if in the experiment the nonlinear effects can be neglected during the time when the different distortions are applied. This will be the case if the characteristic lifetime of the turbulent motion at the entrance of the distorting duct, which is of the order of

$$\tau_l = M/(\tfrac{1}{3}\overline{q_0^2})^{\frac{1}{2}},$$

where M is the mesh length of the grid, is greater than the residence time

$$\tau_r = L/2\overline{U}_1,$$

where L is the length of the distorting duct. In these experiments, the ratio τ_r/τ_l is of the order of 0·2. In an earlier paper (Boschiero, Gence & Mathieu 1977) we have developed a numerical model founded on the simplified Craya equation in which the nonlinear terms have been neglected and comparison with the experimental results appears in figure 7.

An immediate consequence of the relaxation phenomenon is that during the whole time of relaxation the fluctuating motion gives energy to the mean motion. This appears clearly on figure 10 where a strong decay of $\overline{q^2}$ is found after the circular cross-section for $\alpha = \frac{1}{2}\pi$. This is a rather pathological case, but other physical situations exist where this reverse transfer of energy appears, in particular in strongly non-symmetrical flows (Eskinazi 1964). It is also interesting to notice that in the case $\alpha = \frac{1}{2}\pi$ the invariant Π decreases after the change in sign of the strain (figure 7), and then the turbulence presents a tendency to return to isotropy under the action of the mean gradient. The question arises whether in homogeneous turbulence a forced exchange of energy from the fluctuating motion to the mean one is necessarily associated with a forced decay of the invariant Π. In answer we will consider the rate equation for Π, which can be derived easily from the one for the Reynolds stress tensor. If one assumes that

$$2\nu\,\overline{u_{i,k}u_{j,k}} = \tfrac{2}{3}\bar{\epsilon}\delta_{ij}, \tag{16}$$

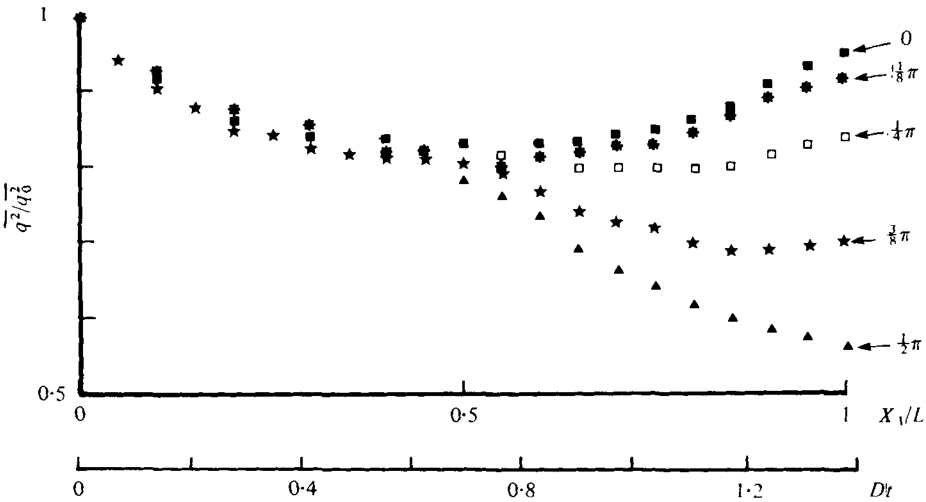

FIGURE 10. Evolution of the ratio $\overline{q^2}/\overline{q_0^2}$ *versus* X_1/L and Dt for different values of α $(0, \frac{1}{8}\pi, \frac{1}{4}\pi, \frac{3}{8}\pi, \frac{1}{2}\pi)$. $\overline{q^2}$ is the trace of the Reynolds stress tensor and $\overline{q_0^2}$ its value at the entrance of the distorting duct.

the evolution equation for Π is

$$\frac{\partial \Pi}{\partial t} = -\tfrac{2}{3}(\overline{U}_{i,j}+\overline{U}_{j,i})\,b_{ij} - 2.(\overline{U}_{i,k}b_{kj}+\overline{U}_{j,k}b_{ki}).b_{ij} + 4.\overline{U}_{i,j}b_{ij}\,\Pi + 4.\bar{\epsilon}/\overline{q^2}.\Pi + 2.\frac{b_{ij}}{\overline{q^2}}\,\overline{\frac{P}{\rho}(U_{i,j}+u_{j,i})}. \quad (17)$$

Of course, as usual in turbulence, this equation must be closed and in particular we can use, for the correlation term involving the pressure, the closure proposed by Lumley (1975):

$$\overline{\frac{P}{\rho}.(u_{i,j}+u_{j,i})} = \frac{\overline{q^2}}{5}(\overline{U}_{i,j}+\overline{U}_{j,i}) + \psi_{ij}(b_{lm}). \quad (18)$$

ψ_{ij} is linear with respect to the components b_{lm}. Since these components are small, we can neglect in (17) all terms nonlinear in b_{ij} and it becomes a very simple equation:

$$\frac{\partial \Pi}{\partial t} \approx -\tfrac{4}{15}(\overline{U}_{i,j}+\overline{U}_{j,i})\,b_{ij}. \quad (19)$$

This can be compared with the rate equation for $\overline{q^2}$ which in the homogeneous case exactly reduces to

$$\frac{\partial \overline{q^2}}{\partial t} = -2.\overline{q^2}.(\overline{U}_{i,j}+\overline{U}_{j,i}).b_{ij} - 2.\epsilon. \quad (20)$$

It is now clear that, when the mean flow receives energy from the fluctuating motion, the double contraction

$$(\overline{U}_{i,j}+\overline{U}_{j,i}).b_{ij} \quad (21)$$

is positive and then Π must decay. The inverse is also true. The experimental results agree well with those coming from the numerical model previously mentioned. Hence we can state that when a mean gradient is applied to a fluctuating motion its energy level and its anisotropy do not necessarily increase but on the contrary can decrease, the evolution of the phenomenon depending of course on the history of the fluctuating motion.

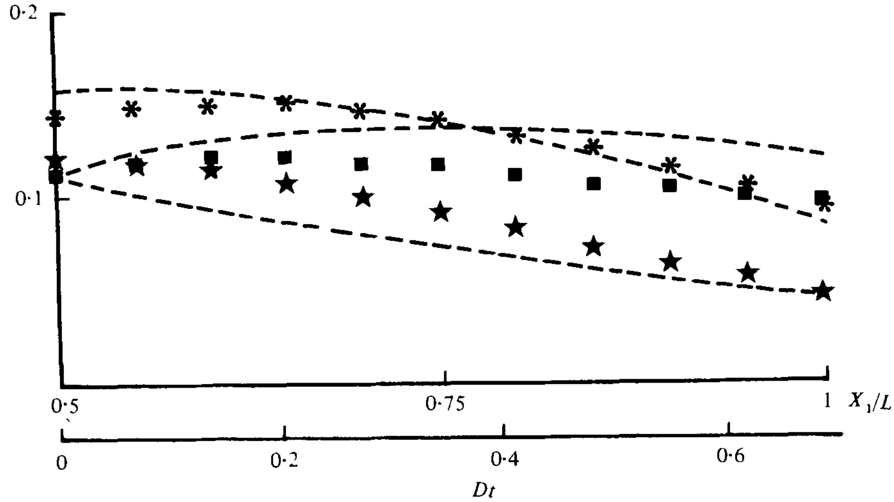

FIGURE 11. Evolution for different values of α ($\frac{1}{8}\pi$, $\frac{1}{4}\pi$, $\frac{3}{8}\pi$) of the component $b_{2'3'}$ as a function of X_1/L and Dt. The origin of time is taken when the turbulence enters the second strain. ---, numerical results of Boschiero, Gence & Mathieu (1977). ■, $\frac{3}{8}\pi$; ∗, $\frac{1}{4}\pi$; ★, $\frac{1}{8}\pi$.

This pathological exchange of energy from the fluctuating motion to the mean flow can be observed not only for $\alpha = \frac{1}{2}\pi$ but also in fact for $\alpha > \frac{1}{4}\pi$. Indeed, the term describing the exchange of energy can be written, *immediately* after the change in position of the principal axes of the strain,

$$\underline{-D.\overline{q^2}.(b_{22}-b_{33})}.(2\cos^2\alpha-1). \tag{22}$$

Since at this location the underlined quantity is positive, this relation will be negative for $\alpha > \frac{1}{4}\pi$ and hence the fluctuating motion will give energy to the mean one, as can be observed in figure 10. Of course we must also note a decay of the invariant II for these values of α, as indicated in figure 7.

Evidently, for the values of α between 0 and $\frac{1}{2}\pi$, the principal axes of the Reynolds stress tensor will not remain aligned with those of the first strain and will rotate around the x_1 axis under the influence of the second strain. In particular, if we work in the principal axes of the new strain $(1, 2', 3')$ as indicated in figure 4, the Reynolds stress tensor is given by

$$\begin{bmatrix} \overline{u_1^2} & 0 & 0 \\ 0 & \overline{u_{2'}^2} & \overline{u_{2'}u_{3'}} \\ 0 & \overline{u_{2'}u_{3'}} & \overline{u_{3'}^2} \end{bmatrix}. \tag{23}$$

Of course, if $\overline{u_1^2}, \overline{u_2^2}, \overline{u_3^2}$ are the components of the Reynolds stress tensor in the axes $(1, 2, 3)$ immediately before the change of strain, we have, just after, in the axes $(1, 2', 3')$

$$\overline{u_{2'}u_{3'}} = \frac{\overline{u_3^2}-\overline{u_2^2}}{2}\sin 2\alpha, \tag{24}$$

or, if the deviator b_{ij} is considered,

$$b_{2'3'} = \frac{b_{33}-b_{22}}{2}\sin 2\alpha. \tag{25}$$

The evolution of this off-diagonal component of the tensor **b** in the axes $(1, 2', 3')$, starting from the value given by the relation (25), appears in figure 11. It is clear that

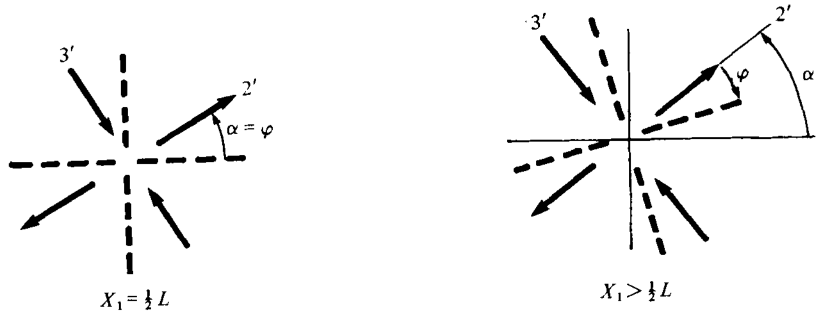

FIGURE 12. The orientation of the principal axes of the Reynolds stress tensor (----).

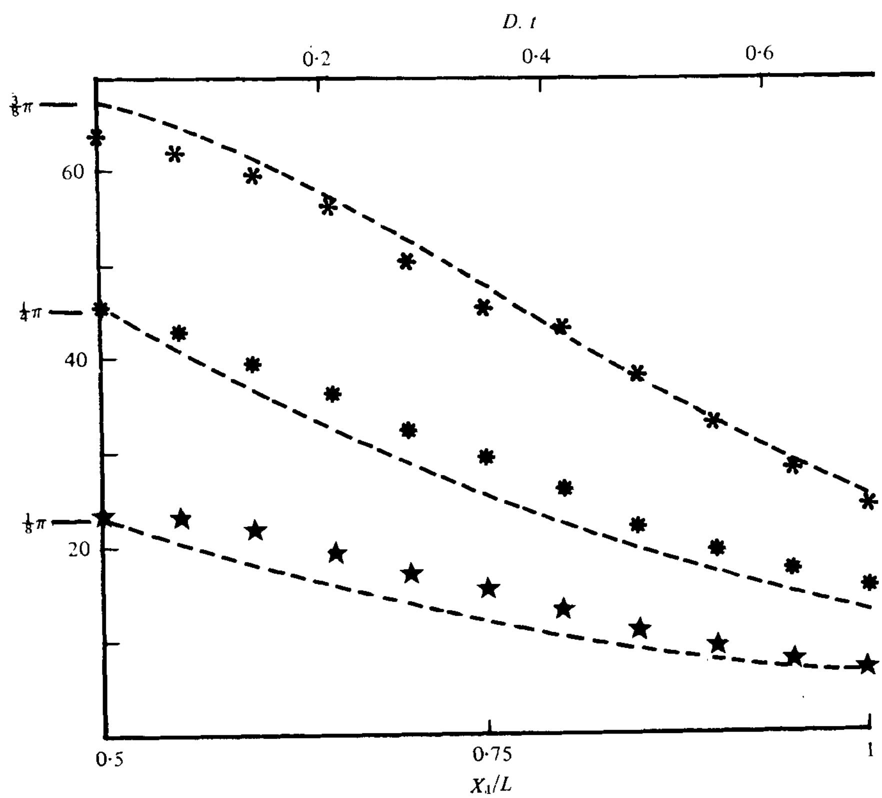

FIGURE 13. Evolution of the angle φ (in degrees) as a function of X_1/L and Dt for different values of α ($\frac{1}{8}\pi$, $\frac{1}{4}\pi$, $\frac{3}{8}\pi$). The origin of time is taken when the turbulence enters the second strain. ---, numerical results of Boschiero, Gence & Mathieu (1977).

the variation of this quantity is very weak. Of course the evolution of $b_{2'3'}$ is not an indication of the evolution of the 'orientation' of the fluctuating motion. More precisely, we should consider the orientation of the principal axes of the Reynolds stress tensor, which are the same as those of **b**, in the principal axes of the new strain. The various notations appear in figure 12. The variations of φ are plotted in figure 13 and it is clear that, at the end of the distorting duct, φ has decayed to half of its initial value. If we define as the characteristic relaxation time, the time τ when φ is half of its initial value, we see that

$$\tau \approx 0{\cdot}7/D, \tag{26}$$

which agrees well with the numerical model of the simplified Craya equation.

Hence, we can say that this relaxation time is of the order of the characteristic time of the strain.

4. Conclusion

We have shown that in homogeneous turbulence associated with a constant mean velocity gradient, an exchange of energy from the fluctuating motion to the mean one is necessarily linked to a forced decay of the anisotropy of the fluctuating motion.

Moreover, when an 'oriented' turbulence is subjected to a plane strain, whose principal axes are different from those of the Reynolds stress tensor, the latter present a tendency to be reoriented in the axes of the strain. The characteristic time of the relaxation phenomenon is of the order of the time scale of the strain. This result can be used to explain the difference between the principal axes of the strain and those of the Reynolds stress tensor in a shear flow (figure 1). Indeed, the principal axes of the Reynolds stress tensor would be aligned with those of the associated strain if the relaxation time τ was much smaller than the characteristic time of rotation of the principal axes of the strain. Now if the time scale of the shear is $1\,s^{-1}$, the characteristic time of both the mean rotation and the associated strain is $2\,s^{-1}$. We know from our previous analysis that the relaxation time scale of the principal axes of the Reynolds stress tensor is then of the order of $2\,s^{-1}$, that is to say of the same order as the mean rotation time. Hence, in a shear, the principal axes of the Reynolds stress tensor cannot be aligned with those of the associated strain which make an angle of $\frac{1}{4}\pi$ with the direction of the flow. It is then clear that the relation between the Reynolds stress tensor and the mean velocity gradient does not satisfy the principle of material indifference as has been argued by Lumley (1970) using other considerations.

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